# The Evolution of Curry 

From Protozoon to Mammal
From Prolog to Haskell

# Eurry <br> funstional <br> logist ${ }^{\text {roggramming }}$ 

## The Best Genes of Both Parents



- functional (algebraic data types, higher order, laziness)
- logic (non-determinism, narrowing)

Why do You need Non-Determinism / Narrowing?

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- logic puzzles (wolf, sheep and cabbage)


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- togic puzzles (wolf, sheep and cabbage)
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## Why do You need Non-Determinism / Narrowing?

- togic puzzles (wolf, sheep and cabbage)
- enumeration of test cases (SmallCheck)
- efficient enumeration of test cases (Lazy SmallCheck)
- circuit analysis (Wired)


## Why do You need Non-Determinism / Narrowing?

- togic puzzles (wolf, sheep and cabbage)
- enumeration of test cases (SmallCheck)
- efficient enumeration of test cases (Lazy SmallCheck)
- circuit analysis (Wired)
- <insert your idea here>


## The first of its Kind

characteristics PAKCS

- Portland Aachen Kiel Curry System
- Michael Hanus
- non-determinism, narrowing
- target language: Prolog
- search strategy: depth first


## Non-Determinism

- overlapping rules induce non-determinism

```
coin :: Int
coin = 0
coin = 1
```


## Non-Determinism

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```
coin :: Int
coin = 0
coin = 1
Main> coin
```


## Non-Determinism

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```
coin :: Int
coin = 0
coin = 1
Main> coin
O
More?
```


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More?
```


## Non-Determinism

- overlapping rules induce non-determinism

```
coin :: Int
coin = 0
coin = 1
Main> coin
O
More?
1
More?
No more Solutions
```


## Non-Determinism

- overlapping rules induce non-determinism

```
coin :: Int
coin = 0
coin = 1
    (?) :: a -> a -> a
    x ? _ = x
_ ? y = y
Main> coin
O
More?
1
More?
No more Solutions
```


## Non-Determinism

- overlapping rules induce non-determinism

```
coin :: Int
coin = 0
coin = 1
Main> coin
O
More?
1
More?
No more Solutions
```

(?) :: a -> a -> a

$$
\mathrm{x} \text { ? _ }=\mathrm{x}
$$

_ ? y = y
coin' :: Int

$$
\operatorname{coin}^{\prime}=0 \text { ? } 1
$$

## Functional Species

double :: Int -> Int<br>double $x=x+x$<br>call-by-value


call-by-name
call-by-need

## Functional Species

```
double :: Int -> Int
double x = x+x
```

call-by-value
call-by-name
double $(0+1)=$ double 1
call-by-need

## Functional Species

$$
\begin{aligned}
& \text { double :: Int -> Int } \\
& \text { double } x=x+x
\end{aligned} \begin{aligned}
\text { call-by-value } \\
\text { double } \begin{aligned}
(0+1) & = \\
& =1+1 \\
& =1
\end{aligned}
\end{aligned}
$$


call-by-name
call-by-name
call-by-need

## Functional Species

$$
\begin{aligned}
& \text { double :: Int -> Int } \\
& \text { double } \mathrm{x}=\mathrm{x}+\mathrm{x}
\end{aligned} \begin{array}{r}
\text { call-by-value } \\
\begin{aligned}
\text { double }(0+1) & =\text { double } 1 \\
& =1+1 \\
& =2
\end{aligned}
\end{array}
$$


call-by-name
call-by-need

## Functional Species

```
double :: Int -> Int
double x = x+x
```

call-by-value

$$
\text { double } \begin{aligned}
(0+1) & =\text { double } 1 \\
& =1+1 \\
& =2
\end{aligned}
$$

call-by-need

call-by-name

## Functional Species

$$
\begin{aligned}
& \text { double : : Int -> Int } \\
& \text { double } \mathrm{x}=\mathrm{x}+\mathrm{x} \\
& \text { call-by-value } \\
& \text { double } \begin{aligned}
(0+1) & =\text { double } 1 \\
& =1+1 \\
& =2
\end{aligned}
\end{aligned}
$$

call-by-name

$$
\text { double } \begin{aligned}
(0+1) & =(0+1)+(0+1) \\
& =1+(0+1)
\end{aligned}
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call-by-name

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\text { double } \begin{aligned}
(0+1) & =(0+1)+(0+1) \\
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\end{aligned}
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## call-by-name

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\begin{aligned}
\text { double }(0+1) & =(0+1)+(0+1) \\
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call-by-need
double $(0+1)=$ let $x=0+1$ in $x+x$

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\end{aligned}
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call-by-need

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\text { double } \begin{aligned}
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\end{aligned}
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& \\
&
\end{aligned}
\end{aligned}
$$



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& =1+1 \\
& =2
\end{aligned}
$$

## Functional Logic Species

$$
\begin{aligned}
& \operatorname{coin}:: \text { Int } \\
& \operatorname{coin}=0 \\
& \operatorname{coin}=1
\end{aligned}
$$

call-by-value
call-by-name

## Functional Logic Species

$$
\begin{aligned}
& \text { coin }:: \text { Int } \\
& \text { coin }=0 \\
& \text { coin }=1
\end{aligned}
$$

call-by-value
double coin $=$ double ( $0 \mid 1$ )
call-by-name

## Functional Logic Species

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\begin{aligned}
& \operatorname{coin}:: \text { Int } \\
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call-by-value

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\text { double coin } & =\text { double }(0 \mid 1) \\
& =\text { double } 0 \mid \text { double } 1
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$$

call-by-name

## Functional Logic Species

```
coin :: Int
coin = 0
coin = 1
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call-by-value

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\text { double coin } & =\text { double }(0 \mid 1) \\
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\end{aligned}
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$\Longrightarrow$ call-time choice
call-by-name

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\end{aligned}
$$

$\Longrightarrow$ call-time choice
call-by-name
double coin = coin+coin

## Functional Logic Species

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coin :: Int
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coin = 1
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call-by-value

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\text { double coin } & =\text { double }(0 \mid 1) \\
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$\Longrightarrow$ call-time choice
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$$
\begin{aligned}
\text { double coin } & =\text { coin+coin } \\
& =(0 \mid 1)+(0 \mid 1)
\end{aligned}
$$

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\begin{aligned}
\text { double coin } & =\text { coin+coin } \\
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& =\ldots=0|1|| | \mid
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\end{aligned}
$$

$\Longrightarrow$ run-time choice

## Functional Logic Species

```
coin : : Int
coin \(=0\)
coin \(=1\)
```

call-by-need

## Functional Logic Species

```
coin :: Int
coin = 0
coin = 1
```

call-by-need
double coin $=$ let $x=$ coin in $x+x$

## Functional Logic Species

```
coin :: Int
coin = 0
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\begin{aligned}
\text { double coin } & =\text { let } x=\operatorname{coin} \text { in } x+x \\
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& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

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coin :: Int
coin = 0
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call-by-need

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& =\text { let } x=0 \mid 1 \text { in } x+x \\
& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

$\Longrightarrow$ run-time choice

## Functional Logic Species

```
coin :: Int
coin = 0
coin = 1
```

call-by-need

$$
\begin{aligned}
\text { double coin } & =\text { let } x=\operatorname{coin} \text { in } x+x \\
& =\text { let } x=0 \mid 1 \text { in } x+x \\
& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

$\Longrightarrow$ run-time choice
double coin $=$ let $x=0 \mid 1$ in $x+x$

## Functional Logic Species

```
coin :: Int
coin = 0
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call-by-need

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& =\ldots=0|1| 1 \mid 2
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\text { double coin } & =\text { let } x=0 \mid 1 \text { in } x+x \\
& =\text { let } x=0 \text { in } x+x \text { | let } x=1 \text { in } x+x
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& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

$\Longrightarrow$ run-time choice

$$
\begin{aligned}
\text { double coin } & =\text { let } x=0 \mid 1 \text { in } x+x \\
& =\text { let } x=0 \text { in } x+x \text { | let } x=1 \text { in } x+x \\
& =0+0 \text { | } 1+1
\end{aligned}
$$

## Functional Logic Species

```
coin :: Int
coin = 0
coin = 1
```

call-by-need

$$
\begin{aligned}
\text { double coin } & =\text { let } x=\operatorname{coin} \text { in } x+x \\
& =\text { let } x=0 \mid 1 \text { in } x+x \\
& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

$\Longrightarrow$ run-time choice

$$
\begin{aligned}
\text { double coin } & =\text { let } x=0 \mid 1 \text { in } x+x \\
& =\text { let } x=0 \text { in } x+x \mid \text { let } x=1 \text { in } x+x \\
& =0+0 \mid 1+1 \\
& =0 \mid 2
\end{aligned}
$$

## Functional Logic Species

```
coin :: Int
coin = 0
coin = 1
```

call-by-need

$$
\begin{aligned}
\text { double coin } & =\text { let } x=\operatorname{coin} \text { in } x+x \\
& =\text { let } x=0 \mid 1 \text { in } x+x \\
& =(0 \mid 1)+(0 \mid 1) \\
& =\ldots=0|1| 1 \mid 2
\end{aligned}
$$

$\Longrightarrow$ run-time choice

$$
\begin{aligned}
\text { double coin } & =\text { let } x=0 \mid 1 \text { in } x+x \\
& =\text { let } x=0 \text { in } x+x \mid \text { let } x=1 \text { in } x+x \\
& =0+0 \text { | } 1+1 \\
& =0 \text { | } 2
\end{aligned}
$$

$\Longrightarrow$ call-time choice

## Non-Determinism

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```


## Non-Determinism

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

Main> insertnd 3 [1,2]

## Non-Determinism

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

Main> insertnd 3 [1,2]
$[1,2,3]$

## Non-Determinism

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
Main> insertnd 3 [1,2]
\([1,2,3]\)
\([1,3,2]\)
```


## Non-Determinism

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
Main> insertnd 3 [1,2]
[1,2,3]
[1,3,2]
[3,1,2]
```


## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```


## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]

## Non-Determinism

permute :: [a] -> [a]
permute []$=[]$
permute $(x: x s)=$ insertnd $x$ (permute $x s$ )

Main> permute [1..3]
$[3,2,1]$

## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]
$[3,2,1]$
$[3,1,2]$

## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd \(x\) (permute \(x s\) )
```

Main> permute [1..3]
$[3,2,1]$
$[3,1,2]$
$[2,3,1]$

## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]
$[3,2,1]$
$[3,1,2]$
$[2,3,1]$
$[2,1,3]$

## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]
$[3,2,1]$
$[3,1,2]$
$[2,3,1]$
$[2,1,3]$
$[1,3,2]$

## Non-Determinism

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]
$[3,2,1]$
$[3,1,2]$
$[2,3,1]$
$[2,1,3]$
$[1,3,2]$
$[1,2,3]$

## Non-Determinism

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
    where
    ys = permute xs
```


## Non-Determinism

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
    where
    ys = permute xs
```

Main> sort $[3,2,1]$

## Non-Determinism

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
    where
    ys = permute xs
```

Main> sort [3,2,1]
$[1,2,3]$

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

Main> list

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

Main> list
[1]

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

Main> list
[1]
[_a,1]

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

Main> list
[1]
[_a,1]
[_a,_b,1]

## Narrowing

list :: [Int]
list = ys ++ [1]
where
ys free

Main> list
[1]
[_a,1]
[_a,_b,1]
[_a,_b,_c,1]

## Narrowing

$$
\begin{aligned}
& \text { last :: [a] -> a } \\
& \text { last xs | ys ++ [y] =:= xs = y } \\
& \text { where } \\
& \text { ys, y free }
\end{aligned}
$$

## Narrowing

$$
\begin{aligned}
& \text { last :: [a] -> a } \\
& \text { last xs | ys ++ [y] == xs = y } \\
& \text { where } \\
& \text { ys, y free }
\end{aligned}
$$

## Narrowing

$$
\begin{aligned}
& \text { last :: [a] -> a } \\
& \text { last xs | ys ++ [y] == xs = y } \\
& \text { where } \\
& \text { ys, y free }
\end{aligned}
$$

Main> last [1..4]

## Narrowing

$$
\begin{aligned}
& \text { last :: [a] -> a } \\
& \text { last xs | ys ++ [y] == xs = y } \\
& \text { where } \\
& \text { ys, y free }
\end{aligned}
$$

Main> last [1..4]
4

## Narrowing

$$
\begin{aligned}
& \text { last :: [a] -> a } \\
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ERROR: '_I_'

## Narrowing Non-Determinism

data Peano = Zero | Succ Peano

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data Peano = Zero | Succ Peano
peano :: Peano
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data Peano = Zero | Succ Peano
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pList = []
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## Narrowing Non-Determinism

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pList = peano : pList
last :: [Peano] -> Peano
last xs | ys ++ [y] == xs = y
    where
\[
\begin{aligned}
& \text { ys = pList } \\
& \text { y = peano }
\end{aligned}
\]
```


## An Evolution Step

characteristics KiCS

- Kiel Curry System
- Bernd Braßel, Frank Huch
- non-determinism
- target language: Haskell
- search strategy: arbitrary, explicit searchtree
- On a Tighter Integration of Functional and Logic Programming, APLAS 2007


## Killer Application

$$
\begin{aligned}
& \text { prop_Insert :: Peano -> [Peano] -> Bool } \\
& \text { prop_Insert p ps = ordered (insert p ps) }
\end{aligned}
$$

## Killer Application

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prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
    where
        x, y free
```


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(Zero,(Succ _a:Zero:_b))
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```


## Haskell ND vs. Curry ND

Haskell ND<br>data Peano = Zero | Succ Peano<br>data [a] = [] | a : [a]<br>\{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...\}

## Haskell ND vs. Curry ND

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data Peano = Zero | Succ Peano
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Curry ND
data Peano = Zero | Succ {Peano}
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{ _ }
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```

characteristics explicit-sharing

- library, available via hackage
- Sebastian Fischer, Oleg Kiselyov, Chung-chieh Shan
- non-determinism
- "target language": monadic Haskell
- search strategy: arbitrary, any MonadPlus

- Purely Functional Lazy Non-deterministic Programming, ICFP 2009
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## this is not the end of evolution!

## Advertisement



The Flavor in Your Programming

PAKCS www.informatik.uni-kiel.de/~pakcs KiCS www.informatik.uni-kiel.de/prog/ mitarbeiter/bernd-brassel/projects explicit sharing sebfisch.github.com/explicit-sharing

> give it a try!

## have

fun!

## have logical fun!

