# The Evolution of Curry

From Protozoon to Mammal

From Prolog to Haskell



## The Best Genes of Both Parents



- functional (algebraic data types, higher order, laziness)
- logic (non-determinism, narrowing)

logic puzzles (wolf, sheep and cabbage)

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- enumeration of test cases (SmallCheck)

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- circuit analysis (Wired)

- logic puzzles (wolf, sheep and cabbage)
- enumeration of test cases (SmallCheck)
- efficient enumeration of test cases (Lazy SmallCheck)
- circuit analysis (Wired)
- <insert your idea here>

# The first of its Kind

#### characteristics PAKCS

- Portland Aachen Kiel Curry System
- Michael Hanus
- non-determinism, narrowing
- target language: Prolog
- search strategy: depth first



```
coin :: Int
coin = 0
coin = 1
```

overlapping rules induce non-determinism

```
coin :: Int
coin = 0
coin = 1
```

Main> coin

```
coin :: Int
coin = 0
coin = 1
Main> coin
0
More?
```

```
coin :: Int
coin = 0
coin = 1
Main> coin
0
More?
1
More?
```

```
coin :: Int
coin = 0
coin = 1
Main> coin
0
More?
1
More?
No more Solutions
```

```
coin :: Int (?) :: a -> a -> a
coin = 0 x ? _ = x
coin = 1 _____? y = y
Main> coin _____? v = y
Main> coin _____? v = y
More?
1
More?
No more Solutions
```

double :: Int -> Int
double x = x+x

call-by-value



double :: Int -> Int
double x = x+x

call-by-value
double (0+1) = double 1



double :: Int -> Int
double x = x+x

call-by-value
double (0+1) = double 1

= 1+1



double :: Int -> Int
double x = x+x



double :: Int -> Int
double x = x+x



double :: Int -> Int
double x = x+x



double :: Int -> Int double x = x+x

call-by-name double (0+1) = (0+1) + (0+1) = 1 + (0+1) = 1 + 1

double :: Int -> Int double x = x+x

call-by-name double (0+1) = (0+1) + (0+1) = 1 + (0+1) = 1 + 1 = 2

double :: Int -> Int
double x = x+x

call-by-need
double (0+1) = let x=0+1 in x+x

double :: Int -> Int double x = x+x

call-by-value
double (0+1) = double 1
 = 1+1
 = 2

double :: Int -> Int
double x = x+x

double :: Int -> Int
double x = x+x

coin :: Int
coin = 0
coin = 1

call-by-value



coin :: Int
coin = 0
coin = 1



call-by-value

double coin = double (0|1)

coin :: Int
coin = 0
coin = 1



#### 

coin :: Int
coin = 0
coin = 1

# 00

#### 

coin :: Int
coin = 0
coin = 1

#### call-by-value

#### double coin = double (0|1) = double 0 | double 1 = 0+0 | 1+1 = 0 | 2



coin :: Int
coin = 0
coin = 1

#### call-by-value

#### double coin = double (0|1) = double 0 | double 1 = 0+0 | 1+1 = 0 | 2

 $\Longrightarrow$  call-time choice



coin :: Int
coin = 0
coin = 1

#### call-by-value

```
double coin = double (0|1)
= double 0 | double 1
= 0+0 | 1+1
= 0 | 2
```

 $\Longrightarrow$  call-time choice

call-by-name

double coin = coin+coin


coin :: Int
coin = 0
coin = 1

#### call-by-value

```
double coin = double (0|1)
= double 0 | double 1
= 0+0 | 1+1
= 0 | 2
```

 $\Longrightarrow$  call-time choice

call-by-name

double coin = coin+coin = (0|1)+(0|1)



coin :: Int
coin = 0
coin = 1

#### call-by-value

```
double coin = double (0|1)
= double 0 | double 1
= 0+0 | 1+1
= 0 | 2
```

```
\Longrightarrow call-time choice
```

#### call-by-name

double coin = coin+coin = (0|1)+(0|1)= ... = 0 | 1 | 1 | 2



coin :: Int
coin = 0
coin = 1

#### call-by-value

```
double coin = double (0|1)
= double 0 | double 1
= 0+0 | 1+1
= 0 | 2
```

```
\Longrightarrow call-time choice
```

#### call-by-name

```
double coin = coin+coin
= (0|1)+(0|1)
= ... = 0 | 1 | 1 | 2
```



coin :: Int
coin = 0
coin = 1

call-by-need



coin :: Int
coin = 0
coin = 1



call-by-need

double coin = let x=coin in x+x

coin :: Int
coin = 0
coin = 1



#### call-by-need

double coin = let x=coin in x+x = let x=0|1 in x+x

coin :: Int
coin = 0
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#### call-by-need

double coin = let x=coin in x+x = let x=0|1 in x+x = (0|1) + (0|1)



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#### call-by-need



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#### call-by-need



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#### call-by-need

 $\implies$  run-time choice

double coin = let x=0|1 in x+x



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coin = 1

#### call-by-need

```
double coin = let x=coin in x+x
 = let x=0|1 in x+x
 = (0|1) + (0|1)
 = ... = 0 | 1 | 1 | 2
```

```
double coin = let x=0|1 in x+x
= let x=0 in x+x | let x=1 in x+x
```



coin :: Int
coin = 0
coin = 1

#### call-by-need

double coin = let x=coin in x+x = let x=0|1 in x+x = (0|1) + (0|1) = ... = 0 | 1 | 1 | 2



coin :: Int
coin = 0
coin = 1

#### call-by-need



coin :: Int
coin = 0
coin = 1

#### call-by-need

 $\implies$  run-time choice

```
double coin = let x=0|1 in x+x
= let x=0 in x+x | let x=1 in x+x
= 0+0 | 1+1
= 0 | 2
```

 $\Longrightarrow$  call-time choice



```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

Main> insertnd 3 [1,2]

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

```
Main> insertnd 3 [1,2] [1,2,3]
```

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

```
Main> insertnd 3 [1,2]
[1,2,3]
[1,3,2]
```

```
insertnd :: a -> [a] -> [a]
insertnd x ys = x:ys
insertnd x (y:ys) = y:insertnd x ys
```

```
Main> insertnd 3 [1,2]
[1,2,3]
[1,3,2]
[3,1,2]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
```

Main> permute [1..3]

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3,2,1]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3,2,1]
[3,1,2]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3,2,1]
[3,1,2]
[2,3,1]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3,2,1]
[3,1,2]
[2,3,1]
[2,1,3]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3,2,1]
[3,1,2]
[2,3,1]
[2,1,3]
[1,3,2]
```

```
permute :: [a] -> [a]
permute [] = []
permute (x:xs) = insertnd x (permute xs)
Main> permute [1..3]
[3, 2, 1]
[3, 1, 2]
[2, 3, 1]
[2, 1, 3]
[1, 3, 2]
[1,2,3]
```

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
where
  ys = permute xs
```

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
where
  ys = permute xs
```

Main> sort [3,2,1]

```
sort :: [a] -> [a]
sort xs | ordered ys = ys
where
  ys = permute xs
Main> sort [3,2,1]
[1,2,3]
```

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

Main> list

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

```
Main> list [1]
```

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

```
Main> list
[1]
[_a,1]
```

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

```
Main> list
[1]
[_a,1]
[_a,_b,1]
```

```
list :: [Int]
list = ys ++ [1]
where
  ys free
```

```
Main> list
[1]
[_a,1]
[_a,_b,1]
[_a,_b,_c,1]
```
```
last :: [a] -> a
last xs | ys ++ [y] =:= xs = y
where
   ys, y free
```

```
last :: [a] -> a
last xs | ys ++ [y] == xs = y
where
    ys, y free
```

```
last :: [a] -> a
last xs | ys ++ [y] == xs = y
where
    ys, y free
```

Main> last [1..4]

```
last :: [a] -> a
last xs | ys ++ [y] == xs = y
where
   ys, y free
```

```
Main> last [1..4]
4
```

```
last :: [a] -> a
last xs | ys ++ [y] == xs = y
where
   ys, y free
```

```
Main> last [1..4]
4
```

Main> last [error "\_|\_",2]

```
last :: [a] -> a
last xs | ys ++ [y] == xs = y
where
   ys, y free
```

```
Main> last [1..4]
4
```

```
Main> last [error "_|_",2]
ERROR: '_|_'
```

data Peano = Zero | Succ Peano

data Peano = Zero | Succ Peano

peano :: Peano peano = Zero peano = Succ peano

```
data Peano = Zero | Succ Peano
```

```
peano :: Peano
peano = Zero
peano = Succ peano
```

```
pList :: [Peano]
pList = []
pList = peano : pList
```

```
data Peano = Zero | Succ Peano
peano :: Peano
peano = Zero
peano = Succ peano
pList :: [Peano]
pList = []
pList = peano : pList
last :: [Peano] -> Peano
last xs | ys ++ [y] == xs = y
 where
  ys = pList
  y = peano
```

## An Evolution Step

#### characteristics KiCS

- Kiel Curry System
- Bernd Braßel, Frank Huch
- non-determinism
- target language: Haskell
- search strategy: arbitrary, explicit searchtree
- On a Tighter Integration of Functional and Logic Programming, APLAS 2007

prop\_Insert :: Peano -> [Peano] -> Bool
prop\_Insert p ps = ordered (insert p ps)

```
prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
```

```
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
where
```

```
x, y free
```

```
prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
```

```
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
where
    x, y free
```

Main> check prop\_Insert

```
prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
```

```
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
where
    x, y free
```

Main> check prop\_Insert
(Zero,(Succ \_a:Zero:\_b))

```
prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
```

```
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
where
    x, y free
```

```
Main> check prop_Insert
(Zero,(Succ _a:Zero:_b))
(Succ Zero,(Succ _a:Zero:_b))
```

```
prop_Insert :: Peano -> [Peano] -> Bool
prop_Insert p ps = ordered (insert p ps)
```

```
check :: (a -> b -> Bool) -> (a,b)
check prop | not (prop x y) = (x,y)
where
    x, y free
```

```
Main> check prop_Insert
(Zero,(Succ _a:Zero:_b))
(Succ Zero,(Succ _a:Zero:_b))
:
```

#### Haskell ND

data Peano = Zero | Succ Peano data [a] = [] | a : [a]

{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

#### **Curry ND**

data Peano = Zero | Succ {Peano}

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

#### **Curry ND**

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

{ \_ }

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

#### **Curry ND**

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

{[], \_:\_ }

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], { _ }:_ }
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], {Zero, Succ _ }:_ }
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], {Zero, Succ { _ }}:_ }
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], {Zero, Succ {Zero, ...}}:_ }
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], {Zero, Succ {Zero, ...}}:{ _ }}
```

#### Haskell ND

```
data Peano = Zero | Succ Peano
data [a] = [] | a : [a]
```

```
{[], [Zero], [Zero,Succ Zero], [Succ Zero,Zero], ...}
```

```
data Peano = Zero | Succ {Peano}
data [a] = [] | {a} : {[a]}
```

```
{[], {Zero, Succ {Zero, ...}}:{[], ...}}
```

#### characteristics explicit-sharing

- library, available via hackage
- Sebastian Fischer, Oleg Kiselyov, Chung-chieh Shan
- non-determinism
- "target language": monadic Haskell
- search strategy: arbitrary, any MonadPlus
- Purely Functional Lazy Non-deterministic Programming, ICFP 2009



#### characteristics explicit-sharing

- library, available via hackage
- Sebastian Fischer, Oleg Kiselyov, Chung-chieh Shan
- non-determinism
- "target language": monadic Haskell
- search strategy: arbitrary, any MonadPlus
- Purely Functional Lazy Non-deterministic Programming, ICFP 2009



## this is not the end of evolution!

### **Advertisement**



PAKCS www.informatik.uni-kiel.de/~pakcs KiCS www.informatik.uni-kiel.de/prog/ mitarbeiter/bernd-brassel/projects explicit sharing sebfisch.github.com/explicit-sharing

## give it a try!

## have

fun!

# have logical fun!